Answers to exam-style questions

Topic 10

Where appropriate, 1 ✓ = 1 mark

1 C

2 C

3 C

4 C

5 C

6 C

7 D

8 B

9 C

10 A

11 **a** The potential at the surface is $V = -\frac{GM}{R} = -5.0 \times 10^{12}\ \mathrm{J\,kg^{-1}}$. ✓

And so $M = -\frac{VR}{G} = \frac{5.0 \times 10^{12} \times 2.0 \times 10^{5}}{6.67 \times 10^{-11}} = 1.5 \times 10^{28}\ \mathrm{kg}$. ✓

b The potential energy at launch on the surface of the planet is mV. ✓

And so the total energy at launch is $\frac{1}{2}mv^2 + mV$. ✓

At the escape speed the total energy has to be zero. ✓
And the result follows.

c $v = \sqrt{-2V} = \sqrt{2 \times 5.0 \times 10^{12}}$ ✓
Which equals $v = 3.2 \times 10^{6}\ \mathrm{m\,s^{-1}}$. ✓

d The work required is $W = m\Delta V$ with $\Delta V = (-1.2 \times 10^{12} - (-5.0 \times 10^{12}) = 3.8 \times 10^{12}\ \mathrm{J\,kg^{-1}}$. ✓
And this is $W = 1500 \times 3.2 \times 10^{12} = 5.7 \times 10^{15}\ \mathrm{J}$. ✓

e The additional energy needed is the kinetic energy: from $\frac{mv^2}{r} = \frac{GMm}{r^2}$ we find $E_K = \frac{1}{2}\frac{GMm}{r} = -\frac{1}{2}mV$ where V is the potential at the position of the probe. ✓

And this is $E_K = -\frac{1}{2} \times 1500 \times (-1.2 \times 10^{12}) = 9.0 \times 10^{14}\ \mathrm{J}$. ✓

f The potential at the release point is $V_1 = -2.2 \times 10^{12}\ \mathrm{J\,kg^{-1}}$ and from conservation of energy

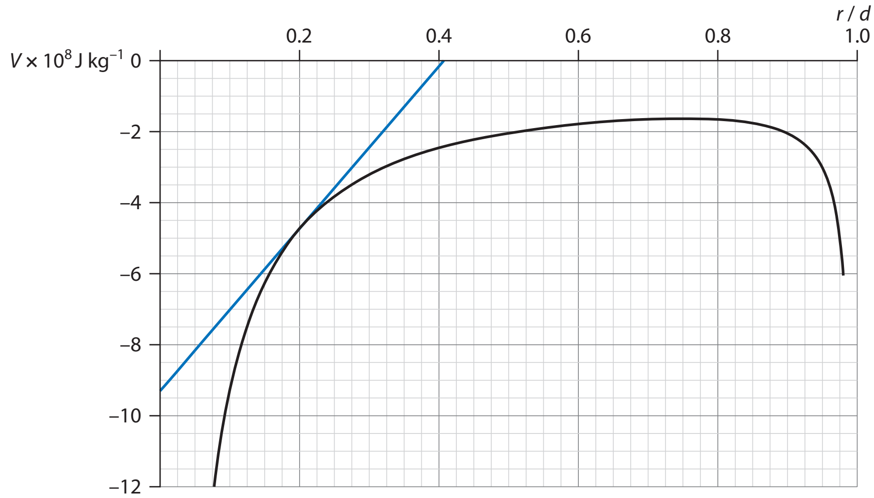
$mV_1 = mV_2 + \frac{1}{2}mv^2$ where is the potential at the surface. ✓

Hence $v = \sqrt{2(V_1 - V_2)} = \sqrt{2(-2.2 \times 10^{12} - (-5.0 \times 10^{12})} = 2.4 \times 10^{6}\ \mathrm{m\,s^{-1}}$. ✓

12 **a** The slope of the tangent is gravitational field strength. ✓

Draw a tangent at the point with $\frac{r}{d} = 0.20$. ✓

Evaluate slope to be $g = \dfrac{0-(-9.2\times10^8)}{(0.41-0)\times4.8\times10^8} \approx 4.7\ \mathrm{N\,kg^{-1}}$. ✓



b The gravitational potential has zero slope there. ✓
Which implies that the gravitational field strength is zero at that point. ✓

c $g = \dfrac{GM}{r_1^2} - \dfrac{Gm}{r_2^2}$ ✓

$0 = \dfrac{GM}{0.75^2} - \dfrac{Gm}{0.25^2}$ ✓

Giving $\dfrac{M}{m} = \dfrac{0.75^2}{0.25^2} = 9.0$ ✓

13 **a** $qV_1 + \frac{1}{2}mv^2 = qV_2$ i.e. $q\dfrac{kQ}{r_1} + \frac{1}{2}mv^2 = q\dfrac{kQ}{r_2}$ ✓

$2.4\times10^{-6}\times\dfrac{8.99\times10^9\times8.8\times10^{-6}}{0.75} + \dfrac{1}{2}\times0.0075\times3.2^2 = 2.4\times10^{-6}\times\dfrac{8.99\times10^9\times8.8\times10^{-6}}{r_2}$ ✓

$0.2532 + 0.3840 (= 0.6372) = \dfrac{0.1899}{r_2}$

Hence $r_2 = 0.2980 \approx 0.30$ m. ✓

b The pellet will move radially away from the sphere. ✓
With an increasing speed but a decreasing acceleration. ✓

c The total energy of the pellet is 0.6372 J and far away this will turn into kinetic energy. ✓

Hence $\frac{1}{2}\times0.075\times v^2 = 0.6372$ J leading to $4.1\ \mathrm{m\,s^{-1}}$. ✓

14 **a** $qV = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\dfrac{2qV}{m}}$ ✓

Hence $v = \sqrt{\dfrac{2\times1.6\times10^{-19}\times29.1}{9.11\times10^{-31}}} = 3.197\times10^6 \approx 3.2\times10^6\ \mathrm{m\,s^{-1}}$. ✓

b The horizontal distance of 2.0 cm is covered at the constant speed found above. ✓

And so $x = vt \Rightarrow t = \dfrac{x}{v} = \dfrac{0.020}{3.197\times10^6} \approx 6.3\times10^{-9}$ s. ✓

c The vertical distance covered is $y = \frac{1}{2}at^2 \Rightarrow a = \frac{2y}{t^2} = \frac{2\times 0.25\times 10^{-2}}{(6.3\times 10^{-9})^2} \approx 1.3\times 10^{14}\ \text{m s}^{-2}$. ✓

And from $qE = ma$ we find $E = \frac{ma}{q} = \frac{9.11\times 10^{-31}\times 1.3\times 10^{14}}{1.6\times 10^{-19}} \approx 740\ \text{N C}^{-1}$. ✓

d The vertical component of velocity at B is $v_y = at = 1.3\times 10^{14}\times 6.3\times 10^{-9} \approx 8.2\times 10^5\ \text{m s}^{-1}$. ✓

Hence $\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{8.2\times 10^5}{3.2\times 10^6} \approx 14°$. ✓

e The work done is the change in kinetic energy. ✓

Which is $\Delta E_K = \frac{1}{2}mv_y^{\ 2} = \frac{1}{2}\times 9.11\times 10^{-31}\times (8.2\times 10^5)^2 = 6.3\times 10^{-17}$ J. ✓

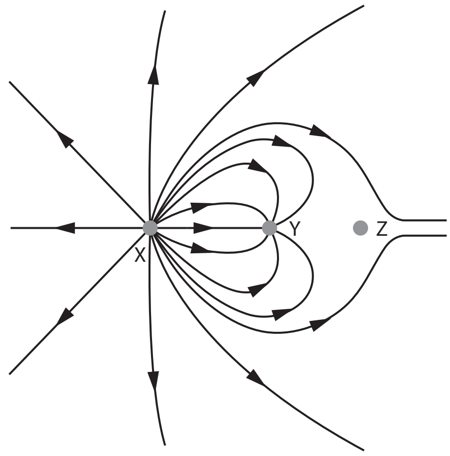
f The work done is also $W = q\Delta V$ and so $\Delta V = \frac{W}{q} = \frac{6.3\times 10^{-17}}{1.6\times 10^{-19}} = 394 \approx 390$ V. ✓

15 a Field lines are mathematical lines originating and ending in electric charges. ✓
Tangents to these lines give the direction of the electric field at a point. ✓

b They leave from positive charges (or infinity) and end in negative charges (or infinity). ✓
They cannot cross. ✓
Their density is proportional to the electric field strength. ✓

c X is positive and Y is negative. ✓

d i The field is zero at a position that may be approximated by Z. ✓



ii The ratio of the distance of Z from X to the distance from Y is about 2.5. ✓

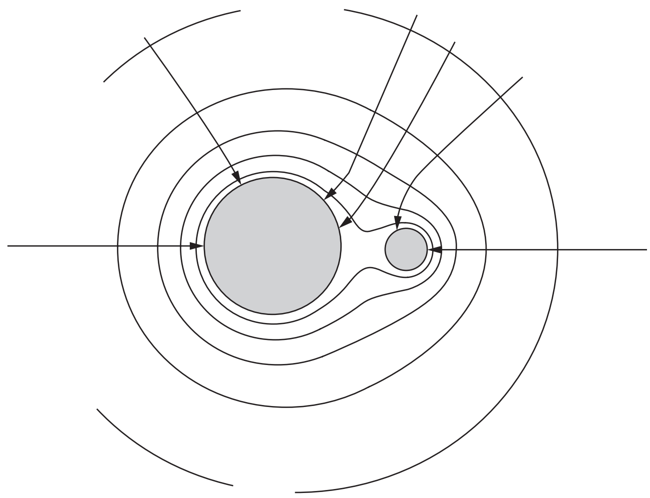
Hence from $0 = \frac{kQ_X}{r_1^2} - \frac{kQ_Y}{r_2^2}$ we find $\frac{Q_X}{Q_Y} = \frac{r_1^2}{r_2^2} = 2.5^2 \approx 6$. ✓

16 a i An equipotential surface is the set of all points that have the same potential. ✓

b i Field lines normal to equipotentials. ✓

And normal to spheres. ✓

(plus symmetrically paced lines one the lower side)



ii The potential difference between A and B is $\Delta V = 2.0 \times 10^6\ \text{J kg}^{-1}$. ✓

And so the work done is $m\Delta V = 1500 \times 2.0 \times 10^6 = 3.0 \times 10^9\ \text{J}$. ✓

iii $g \approx \dfrac{\Delta V}{\Delta r}$ ✓

$g \approx \dfrac{10^6}{4.0 \times 10^6} = 0.25\ \text{N kg}^{-1}$ ✓

iv From a very large distance away the two bodies look like one point particle. ✓

And the equipotential surfaces of a single particle are spherical. ✓

c The potential; lines shown correspond to two masses so they are defined by $-\dfrac{GM_1}{r_1} - \dfrac{GM_2}{r_2} = \text{constant}$, or just

$-\dfrac{M_1}{r_1} - \dfrac{M_2}{r_2} = \text{constant}$. ✓

Two positive charges or two negative charges would give equipotential lines defined by

$-\dfrac{Q_1}{r_1} - \dfrac{Q_2}{r_2} = \text{constant}$. ✓

And so would be the same as in the gravitational case. ✓